## Calculus Warm Up #2-1

Find the limits:

1. $\lim_{x \to 2} \dfrac{x^5 - 32}{x - 2}$

hint: 2

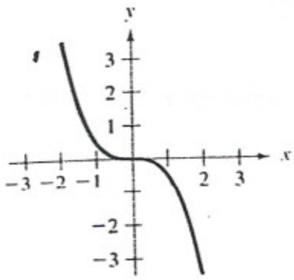
2. $\lim_{x \to 1} \dfrac{\sqrt{x+3} - 2}{x - 1}$

3. $\lim_{x \to -1} \dfrac{\frac{2}{x+3} + \frac{1}{x}}{x + 1}$

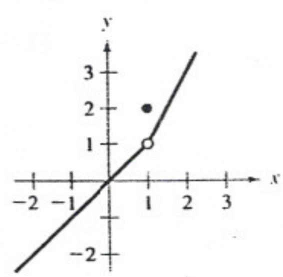
HW Questions:

In Exercises 1–6, find the points of discontinuity (if any).

**1.** $f(x) = -\dfrac{x^3}{2}$




**5.** $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$

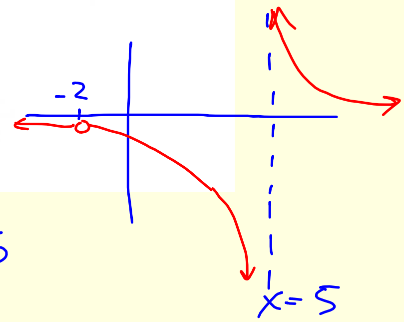


In Exercises 7–24, find the discontinuities (if any) for the given function. Which of the discontinuities are removable?

**9.** $f(x) = \dfrac{1}{x-1}$

**13.** $f(x) = \dfrac{x+2}{x^2 - 3x - 10}$

$(x-5)(x+2)$




discontinuities @ $x = -2$ and $5$

Removable at $x = -2$

Support: $\lim\limits_{x \to -2} \dfrac{x+2}{(x-5)(x+2)}$

$= \lim\limits_{x \to -2} \dfrac{1}{x-5} = -\dfrac{1}{7}$

hole at $(-2, -\frac{1}{7})$

Find discontinuities; which are removable?

**17.** $f(x) = \begin{cases} \dfrac{x}{2} + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

21. $f(x) = \begin{cases} |x - 2| + 3, & x < 0 \\ x + 5, & x \ge 0 \end{cases}$

In Exercises 25–30, discuss the continuity of the composite function $h(x) = f(g(x))$.

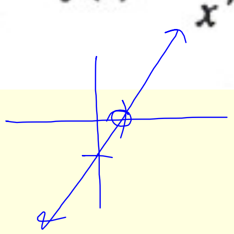
25. $f(x) = x^2$, $g(x) = x - 1$

$$f(g(x)) = \frac{1}{\frac{1}{x-1}}$$

29. $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{1}{x-1}$

$$f(g(x)) = 1 \cdot (x-1)$$

$$f(g(x)) = x - 1$$



$x \neq 1$

$\therefore f(g(x))$ has a removable discontinuity at $(1, 0)$

Sketch the graph of the function and determine any points of discontinuity.

33. $f(x) = [\![x]\!] - x$

find greatest integer function on grapher.

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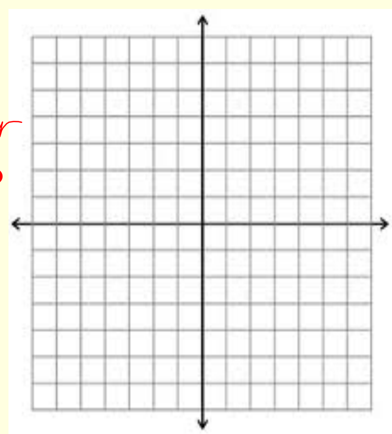


37. Continuous on what interval? (given graph)

$f(x) =$

In Exercises 41 and 42, use the Intermediate Value Theorem to approximate the zero of the given function in the interval [0, 1]. (a) Begin by locating the zero in a subinterval of length 0.1. (b) Refine your approximation by locating the zero in a subinterval of length 0.01.

**41.** $f(x) = x^3 + x - 1$

a) Table set Start = 0
$\Delta$Tbl = 0.1

| 0.6 | 0.7 |
|---|---|
| -0.184 | 0.043 |

$0.6 < \text{zero} < 0.7$

zero is on the interval $(0.6, 0.7)$

b) Start = 0.6
$\Delta$Tbl = 0.01

| 0.68 |
|---|
| |

In Exercises 43–46, verify the applicability of the Intermediate Value Theorem in the indicated interval and find the value of $c$ guaranteed by the theorem.

45. $f(x) = x^3 - x^2 + x - 2$, $[0, 3]$, $f(c) = 4$

All cubics are continuous on $(-\infty, \infty)$

Can apply the Intermediate Value theorem

if $f(0) < f(c) < f(3)$

$-2 < 4 < 19$, so yes

Find c

$4 = x^3 - x^2 + x - 2$

$0 = x^3 - x^2 + x - 6$

$0 = (x-2)(x^2 + x + 3) \rightarrow$ No real x here

$b^2 - 4ac < 0$

$1 - 12 < 0$

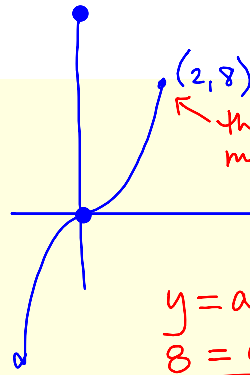
$-11 < 0$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

$\left.\begin{array}{l} x = 2 \\ f(2) = 4 \end{array}\right\}$ $\boxed{c = 2}$

**47.** Determine the constant $a$ so that the following function is continuous on the entire real line.

$$f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$$



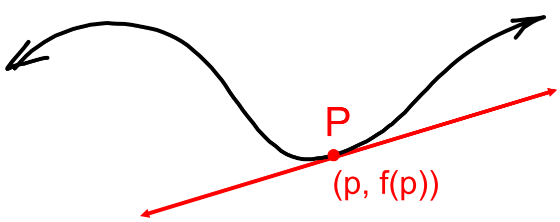

the parabola must start here at $x = 2$ where $y = 8$

$y = ax^2$

$8 = a(2)^2$

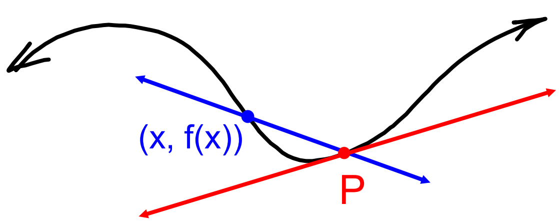
$\boxed{a = 2}$

Finding the equation of a line tangent to a curve.




Point - Slope Form

$y - f(p) = m(x - p)$




$$m = \lim_{x \to p} \frac{f(x) - f(p)}{x - p}$$

You try:

$f(x) = \sqrt{x}$ $\qquad g(x) = \frac{1}{x}$

1. Find the point-slope equation of the line tangent to f(x) through (4, 2).

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$\lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$\frac{1}{4}$$

$y-2=\frac{1}{4}(x-4)$

$g(2)=\frac{1}{2}$

2. Find the line tangent to g(x) where x = 2.

$$\lim_{x\to 2}\left[\frac{\frac{1}{x}-\frac{1}{2}}{x-2}\right]$$

$$=\lim_{x\to 2} \frac{1}{x-2}\left[\frac{2}{2}\cdot\frac{1}{x}-\frac{1}{2}\cdot\frac{x}{x}\right]$$

$$=\lim_{x\to 2} \frac{1}{x-2}\cdot\frac{2-x}{2x}$$

$$=\lim_{x\to 2} \frac{-1\cancel{(x-2)}}{\cancel{(x-2)}(2x)}$$

$$=-\frac{1}{4}$$

$y-\frac{1}{2}=-\frac{1}{4}(x-2)$

2.1-2.3

- Strategies for finding limits
- One sided limits
- Equations for lines tangent to a curve
- Continuity
- The Intermediate Value Theorem

2.4

-Infinite Limits

-Vertical Asymptotes

## Infinite Limits

$$\lim_{x \to c} f(x) = \infty$$

Means:

1) f(x) increases without bound as x approaches c.
   (decreases if the limit = $-\infty$)

2) f(x) has infinite discontinuity at x = c.

Note: By definition, the limit does not exist because the function does not approach a single value, but it is useful to be able to talk about infinite limits.

## Definition of a Vertical Asymptote:

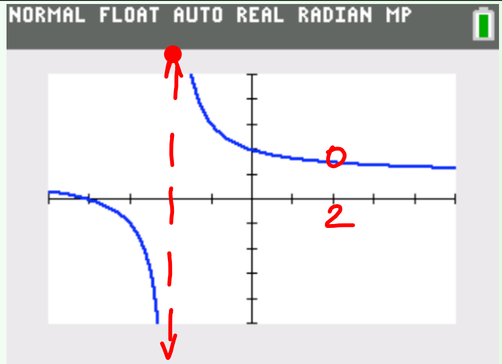
If $\boxed{\lim_{x \to c} f(x) = \pm\infty}$, then there is a vertical asymptote at x = c.

Ex: $f(x) = \dfrac{x^2 + 2x - 8}{\underset{(x+2)(x-2)}{x^2 - 4}}$

We can see from the denominator that f(x) is undefined at x = ± 2.

Does f(x) have vertical asymptotes? Check limits to see if they are infinite.

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 4}$$




Asymptote at x = -2

What about x = 2 ?

$$\lim_{x \to 2} \frac{(x+4)(x-2)}{(x+2)(x-2)}$$

$$\lim_{x \to 2} \frac{x+4}{x+2}$$

$$= \frac{3}{2}$$

→ removable discontinuity at $x = 2$

The point $(2, \frac{3}{2})$ plugs the hole!

Vertical Asymptote Theorem

For $h(x) = \dfrac{f(x)}{g(x)}$

at x = c, if $h(c) = \dfrac{k}{0}$ ⟵(a constant, ≠0)

then x = c is a vertical asymptote.

Note: If $h(c) = \dfrac{0}{0}$ , not a vertical asymptote.

Is it a hole or maybe a gap?

Two ways to confirm a vertical asymptote:

$$f(x)=\frac{x^2+2x-8}{x^2-4}$$

1) Check f(-2):

$$f(-2)=\frac{(-2)^2+2(-2)-8}{(-2)^2-4}$$

$$=\frac{-8}{0} \qquad \frac{k}{0}$$

$\frac{\text{A constant}}{0}$ confirms asymptote

2) Check for infinite limit:

$$\lim_{x\to -2}\frac{(x+4)(x-2)}{(x+2)(x-2)}$$

$$\lim_{x\to -2}\frac{x+4}{x+2}$$

$$=\frac{2}{0}$$

$$=\infty$$

2 ÷ by a super small # close to zero is heading to ∞

How can we tell from the function if the discontinuities are vertical asymptotes or holes?

$$f(x)=\frac{x^2+2x-8}{x^2-4}$$

$$f(x)=\frac{(x+4)(x-2)}{(x+2)(x-2)}$$

discontinuity is a hole if you can cancel the factor

$$g(x)=\frac{x+4}{x+2} \rightarrow$$

g is equivalent to f except that it doesn't have the hole. We <u>removed</u> it!

Determine all vertical asymptotes: (practice)

1. $f(x)=\dfrac{1}{2(x+1)}$

2. $f(x)=\dfrac{x^2+1}{x^2-1}$

3. $f(x)=\dfrac{x^2-3x}{x-1}$

Determine all vertical asymptotes:

1. $f(x)=\dfrac{1}{2(x+1)}$

discontinuity @ $x=-1$

$f(-1)=\frac{1}{0}$ ← confirms vertical asymptote at $x=-1$

2. $f(x)=\dfrac{x^2+1}{x^2-1}$

discontinuities at $x=\pm 1$

$f(\pm 1)=\frac{2}{0}$, so verticals at both $x=\pm 1$

3. $f(x)=\dfrac{x^2-3x}{x-1}$

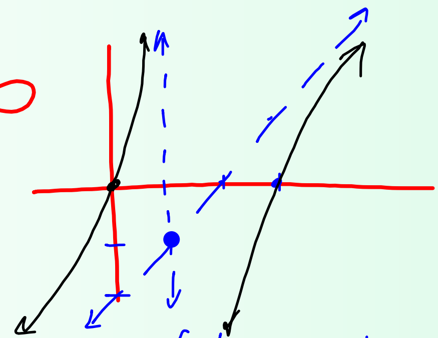
discontinuity at $x=1$

$f(1)=-\frac{2}{0}$, so vertical asymptote @ $x=1$

Find the limits:

1. $\lim_{x\to 1^-} \dfrac{x^2-3x}{x-1} = \infty$

from left



2. $\lim_{x\to 1^+} \dfrac{x^2-3x}{x-1} = -\infty$

from right

degree of top exactly one more than bottom so find slant asymptote:

$$\begin{array}{r|rrr} 1 & 1 & -3 & 0 \\ & & 1 & -2 \\ \hline & 1 & -2 & \end{array}$$

$y = x - 2$

☆ Infinite Limits confirm asymptote @ $x = 1$

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If c and L are real numbers and

$\lim_{x\to c} f(x) = \infty$ $\quad$ $\lim_{x\to c} g(x) = L$

then these properties are true:

1. Sum or difference: $\lim_{x\to c}\left[f(x) \pm g(x)\right] = \infty$

$\lim_{x\to c} f(x) \pm \lim_{x\to c} g(x) = \infty$

$\infty + L = \infty$

2. Quotient: $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$ $\quad$ $\dfrac{\lim g}{\lim f} \Rightarrow \dfrac{k}{\infty} = 0$

3. Product: $\lim_{x\to c}\left[f(x)g(x)\right] = \infty, L > 0$ $\quad$ $(\infty)(L) = \infty$

$\lim_{x\to c}\left[f(x)g(x)\right] = -\infty, L < 0$

Find the limits: (practice)

1. $\lim_{x\to 0}\left(1+\frac{1}{x^2}\right)=$

$f(x)=1$ $g(x)=\frac{1}{x^2}$

2. $\lim_{x\to 1^-}\frac{x^2+1}{1/(x-1)}=$

Find the limits:

1. $\lim_{x\to 0}\left(1+\frac{1}{x^2}\right)= \lim_{x\to 0} 1 + \lim_{x\to 0}\frac{1}{x^2}$

$f(x)=1$ $g(x)=\frac{1}{x^2}$

$= 1 + \frac{1}{0}$

$= 1 + \infty$

$= \infty$

2. $\lim_{x\to 1^-}\frac{x^2+1}{1/(x-1)}= \frac{\lim_{x\to 1^-}(x^2+1)}{\lim_{x\to 1^-}\frac{1}{x-1}} = \frac{2}{\frac{1}{0}}$

$= \frac{2}{\infty}$

$= 0$

# HW: p. 82 # 1 - 41 eoo

HW quiz tomorrow on last week's HW.

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tangent line worksheet

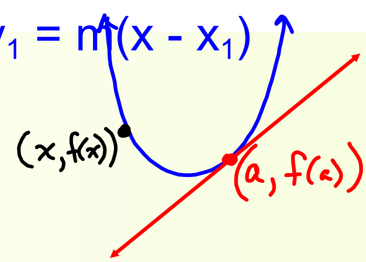
Express all line equations in Point-Slope form:

$y - y_1 = m(x - x_1)$

1. $f(x) = x^2 + 3x + 4$

a) find the slope of the tangent line to $f(x)$ at any point $(a, f(a))$.

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$




$$\lim_{x \to a} \frac{x^2 + 3x + 4 - (a^2 + 3a + 4)}{x - a}$$

$$\lim_{x \to a} \frac{x^2 - a^2 + 3x - 3a}{x - a}$$

factoring top:

$(x + a)(x - a) + 3(x - a)$

$(x - a)(x + a + 3)$

$$\lim_{x \to a} \frac{\cancel{(x - a)}(x + a + 3)}{\cancel{x - a}}$$

$m = a + a + 3$

$m = 2a + 3$ at $(a, f(a))$

b) Find the slope of the tangent at (0,4).

$m = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ or use $m = 2a+3$ for $a = 0$

$\boxed{m = 3}$

c) Find the slope of the tangent where $x = -2$.

$m = \lim_{x \to -2} \frac{f(x) - f(-2)}{x+2}$ or $m = 2(-2)+3$

$\boxed{m = -1}$

5. The tangent line to the parabola $f(x) = x^2 + cx + d$ at the point (1,2) has a slope of 3. Find c.

$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = 3$ and $f(1) = 2$

$1^2 + c(1) + d = 2$

$d = 1 - c$

sub in for d.

$\lim_{x \to 1} \frac{x^2 + cx + 1 - c - 2}{x - 1} = 3$

$\lim_{x \to 1} \frac{x^2 - 1 + cx - c}{x - 1} = 3$

$\lim_{x \to 1} \frac{(x-1)(x+1) + c(x-1)}{x - 1} = 3$

$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1+c)}{\cancel{x-1}} = 3$

$1 + 1 + c = 3$

$\boxed{c = 1}$

AP Calculus (BC): Worksheet #1: Slope of a tangent line.

1. $f(x) = x^2 + 3x + 4$.
   (a) Find the slope predicting formula for the slope of the tangent to the graph of $y = f(x)$ at any point $(a, f(a))$
   (b) Find slope of the tangent at point ((0,4)
   (c) Find slope of the line that is tangent to $y = f(x)$ at $x = -2$.
   (d) Find the equation of the tangent line through (-3, 4).

2. Find the equation of the tangent line to the graph of $f(x) = 3 - x^2$ at the point (-2, -1).

3. Find the equation of the tangent line to the graph of $f(x) = \sqrt{x}$ at the point (4, 2).

4. Find the equation of the line tangent to the graph of $f(x) = \dfrac{1}{x}$ at the point (2, ½).

5. The tangent line to the parabola $f(x) = x^2 + cx + d$ at the point (1, 2) has slope 3. Find $c$